

# Thermodynamics of a field theory with infrared fixed point from gauge/gravity duality

J. Alanen<sup>a,b\*</sup>, K. Kajantie<sup>a,b†</sup>

<sup>a</sup>*Department of Physics, P.O.Box 64, FI-00014 University of Helsinki, Finland*

<sup>b</sup>*Helsinki Institute of Physics, P.O.Box 64, FI-00014 University of Helsinki, Finland*

We use gauge/gravity duality to study the thermodynamics of a field theory with asymptotic freedom in the ultraviolet and a fixed point in the infrared. We find a high temperature quark-gluon phase and a low  $T$  conformal unparticle phase. The phase transition between the phases is of first order or continuous, depending on the ratio of the radii of asymptotic  $\text{AdS}_5$  spaces at  $T = 0$  and  $T = \infty$ . This is a prediction from a model of gauge/gravity duality, not yet verified on the field theory side.

---

\*janne.alanen@helsinki.fi

†keijo.kajantie@helsinki.fi

# 1 Introduction

A gauge/gravity duality model for  $SU(N_c)$  gauge theory thermodynamics with a high temperature gluon plasma phase and a low  $T$  glueball gas phase and a first order transition in between has been presented in [1, 2]. The model contains a 5-dimensional metric ansatz and a scalar field with a bulk potential. A variant of the model has been developed in [3], in which the scalar field potential as a starting point is replaced by the beta function of the boundary field theory.

The purpose of this article is to apply the method in [3] to a boundary field theory with the model beta function [4, 5]

$$\beta(\lambda) = -\beta_0 \lambda^2 \left(1 - \frac{\lambda}{\lambda_*}\right), \quad \lambda = N_c g^2, \quad (1)$$

where  $\beta_0 > 0$  is a parameter and  $\lambda = \lambda_*$  is an infrared (IR) fixed point. What is the finite temperature phase structure of this theory? At high temperatures one expects the existence of a "quark-gluon plasma" phase<sup>1</sup>, conformal at  $T \rightarrow \infty$ . Due to the IR fixed point one knows that there are no massive glueball states in the theory. Thus there is no low  $T$  glueball phase but, instead, a conformal "unparticle" phase of massless particles. We shall show that, indeed, this picture follows naturally from the model in [1, 3]. Furthermore, the transition is of first order for  $\beta_0 \lambda_* > 6.58$  and a crossover for smaller values.

More formally, at  $T = 0$  and  $T = \infty$  we have two pure  $AdS_5$  spaces with radii  $\mathcal{L}_{IR}$  and  $\mathcal{L}_{UV}$  and the transition we study is driven by varying  $T$ . As we shall see, the parameter  $\beta_0$  is essentially the ratio of the radii:

$$\beta_0 \lambda_* = \frac{9}{2} \log \frac{\mathcal{L}_{UV}}{\mathcal{L}_{IR}}. \quad (2)$$

Beta functions of the type in Eq.(1) but somewhat more complicated have recently become a central topic of research in connection of technicolor theories [6, 7, 8, 9] and lattice Monte Carlo studies thereof [10, 11, 12, 13, 14, 15, 16]. The results in this article should set the stage for the study of the thermodynamics of these more general theories. Is it possible to use lattice Monte Carlo to search for the two-phase structure predicted here and to determine the order of the transition?

## 2 The gravity sector of the model

We summarize first the gravity sector equations of the model in [1, 3]. The model starts from a metric ansatz

$$ds^2 = b^2(z) \left[ -f(z) dt^2 + d\mathbf{x}^2 + \frac{dz^2}{f(z)} \right] \quad (3)$$

plus a scalar field  $\phi(z) = \log \lambda(z)$ , where the four functions  $b(z)$ ,  $f(z)$  in the metric, the scalar field  $\phi(z)$  and the potential  $V(\phi(z))$  in the gravity action (in standard notation)

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[ R - \frac{4}{3} (\partial_\mu \phi)^2 + V(\phi) \right] \quad (4)$$

---

<sup>1</sup> Realistic theories of the type (1) have fermions and we assume that we can apply the model in [3] to them.

are determined from the four equations ( $\dot{b} \equiv b'(z)$ , etc.)

$$6\frac{\dot{b}^2}{b^2} + 3\frac{\ddot{b}}{b} + 3\frac{\dot{b}\dot{f}}{bf} = \frac{b^2}{f}V(\phi), \quad (5)$$

$$6\frac{\dot{b}^2}{b^2} - 3\frac{\ddot{b}}{b} = \frac{4}{3}\dot{\phi}^2, \quad (6)$$

$$\frac{\ddot{f}}{f} + 3\frac{\dot{b}}{b} = 0, \quad (7)$$

$$\beta(\lambda) = b\frac{d\lambda}{db}, \quad \lambda(z) = e^{\phi(z)} \sim g^2 N_c. \quad (8)$$

First, from (8) it follows that, for the beta function (1),

$$\log \frac{b(\lambda)}{b_0} = \frac{1}{\hat{\beta}_0} \left[ \frac{1}{\hat{\lambda}} - \log \frac{\hat{\lambda}}{1 - \hat{\lambda}} \right] \equiv Q + \frac{1}{\hat{\beta}_0} \log(\hat{\beta}_0 Q - 1), \quad (9)$$

where we have scaled (hats will be removed after Eq. (16))

$$\hat{\lambda} = \frac{\lambda}{\lambda_*}, \quad \hat{\beta}_0 = \lambda_* \beta_0 \quad (10)$$

and where we have introduced [3]

$$Q \equiv \frac{1}{\hat{\beta}_0 \hat{\lambda}}. \quad (11)$$

Further, to solve the second equation one introduces

$$W = -\dot{b}/b^2, \quad (12)$$

and solves

$$\begin{aligned} W(\lambda) &= W(0) \exp \left( -\frac{4}{9} \int_0^\lambda d\bar{\lambda} \frac{\beta(\bar{\lambda})}{\bar{\lambda}^2} \right) = W(0) e^{\frac{4}{9} \hat{\beta}_0 (\hat{\lambda} - \frac{1}{2} \hat{\lambda}^2)} \\ &= \frac{1}{\mathcal{L}} e^{\frac{2}{9} \hat{\beta}_0} e^{-\frac{2}{9} \hat{\beta}_0 (1 - \hat{\lambda})^2} = \frac{1}{\mathcal{L}} e^{\frac{4}{9Q} - \frac{2}{9\hat{\beta}_0 Q^2}}. \end{aligned} \quad (13)$$

Given  $W$  one can immediately write down [4, 5] the scalar field potential  $V(\phi)$  corresponding to  $f(z) = 1$  in (5) (from now on hats are removed):

$$V(\lambda = e^\phi) = \frac{12}{\mathcal{L}^2} e^{\frac{4}{9}\beta_0} e^{-\frac{4}{9}\beta_0(1-\lambda)^2} \left[ 1 - \frac{1}{9} \beta_0^2 \lambda^2 (1 - \lambda)^2 \right]. \quad (14)$$

In the UV ( $\lambda = 0$ ) and IR ( $\lambda = 1$ ) limits we have two AdS<sub>5</sub> spaces with radii  $\mathcal{L} = \mathcal{L}_{\text{UV}}$  and  $\mathcal{L}_{\text{IR}} = \mathcal{L}_{\text{UV}} e^{-2\beta_0/9}$ . Eq. (2), giving the parameter  $\beta_0$  as essentially their ratio, follows immediately from this.

When  $f$  is the solution of (7), (5) gives the potential

$$V(\lambda) = 12fW^2 \left[ 1 - \left( \frac{\beta}{3\lambda} \right)^2 \right] - 3\frac{\dot{f}}{b}W. \quad (15)$$

From Eq. (12) one further obtains, defining the energy scale  $\Lambda = b_0/\mathcal{L}$ ,

$$\Lambda z = \int_Q^\infty \frac{dQ}{(\beta_0 Q - 1)^{1/\beta_0+1}} \beta_0 Q e^{-Q - \frac{4}{9Q} + \frac{2}{9\beta_0 Q^2}}. \quad (16)$$

Finally, from Eq. (7) we solve

$$f(z) = 1 - \int_0^z \frac{d\bar{z}}{b^3(\bar{z})} / \int_0^{z_h} \frac{d\bar{z}}{b^3(\bar{z})}, \quad (17)$$

where two constants of integration are fixed by  $f(0) = 1$  and  $f(z_h) = 0$ .

Thermodynamics now is obtained from the key relations

$$4\pi T = -f'(z_h), \quad S = \frac{A}{4G_5} = \frac{b^3}{4G_5} V_3. \quad (18)$$

For the temperature we thus have

$$\frac{1}{4\pi T} = b^3 \int_0^z \frac{dz}{b^3} = b^3 \int_0^\lambda \frac{d\lambda}{-\beta b^4 W} = b^3 \int_Q^\infty \frac{dQ}{1 - 1/(\beta_0 Q)} \frac{1}{b^4 W} \quad (19)$$

or

$$\frac{\Lambda}{4\pi T} = e^{3Q} (\beta_0 Q - 1)^{3/\beta_0} \int_Q^\infty \frac{dQ}{(\beta_0 Q - 1)^{4/\beta_0+1}} \beta_0 Q e^{-4Q - \frac{4}{9Q} + \frac{2}{9\beta_0 Q^2}}. \quad (20)$$

The derivative  $dT/dQ$  will be very important in what follows and this is computed to be

$$\frac{dT}{T dQ} = \frac{\beta_0 Q}{\beta_0 Q - 1} \left( \frac{4\pi T}{bW} - 3 \right). \quad (21)$$

Full thermodynamics is then given by

$$s(T) = \frac{1}{4G_5} b^3(Q(T)), \quad (22)$$

$$p(T) = \int_0^T dT s(T) = \frac{1}{4G_5} \int_0^{Q(T)} dQ \frac{dT}{dQ} b^3(Q), \quad (23)$$

$$\epsilon(T) = \frac{3}{4G_5} \int_0^{Q(T)} dQ T(Q) b^3(Q) \frac{d \log b}{dQ}, \quad (24)$$

$$\epsilon(T) - 3p(T) = \frac{3}{4G_5} \int_0^{Q(T)} dQ \left( T \frac{d \log b}{dQ} - \frac{dT}{dQ} \right) b^3(Q), \quad (25)$$

$$c_s^2 = \frac{1}{3T} \frac{dT}{dQ} \frac{dQ}{d \log b} = \frac{1}{3} \left( 4 \frac{\pi T}{bW} - 3 \right) = \frac{s}{C_V}, \quad (26)$$

where  $b(Q)$  is given by (9) and  $T(Q)$  by (20).

### 3 Analytic approximations in the ultraviolet and infrared

Final results will have to be computed numerically, but it is very useful to have analytic approximations in the ends of the phase space, UV ( $Q \rightarrow \infty$ ) and IR ( $Q \rightarrow 1/\beta_0$ ).

The leading UV terms are very simple:

$$\frac{b}{b_0} = e^Q (\beta_0 Q)^{\frac{1}{\beta_0}} = \frac{1}{\Lambda z} \equiv \frac{\mathcal{L}}{b_0 z} = \frac{\pi T}{\Lambda}; \quad (27)$$

further corrections can be easily evaluated. The UV limit of bulk thermodynamic quantities is easiest to obtain from

$$\frac{s}{T^3} = \frac{\mathcal{L}^3}{4G_5} \frac{(b(Q \rightarrow \infty)/b_0)^3}{(T(Q \rightarrow \infty)/\Lambda)^3} = \frac{\mathcal{L}^3}{4G_5} \pi^3 \quad (28)$$

since the UV limit here is nothing but  $1/(z_h T)^3 = \pi^3$ . Equivalently,

$$\frac{p}{T^4} = \frac{\mathcal{L}^3}{4G_5} \frac{\pi^3}{4}, \quad T \rightarrow \infty. \quad (29)$$

Normalising to ideal gas pressure  $p = g_{\text{eff}} \pi^2 T^4 / 90$  one has

$$\frac{\mathcal{L}^3}{4G_5} = \frac{2}{45\pi} g_{\text{eff}}. \quad (30)$$

The conformal IR region is the region near  $\beta_0 Q - 1 = 0$  where the powerlike behavior of  $b(Q)$  drives it to zero. For  $b$  and  $W$  one can trivially expand:

$$b = b_0 e^{\frac{1}{\beta_0}} (\beta_0 Q - 1)^{\frac{1}{\beta_0}} \left[ 1 + \frac{1}{\beta_0} (\beta_0 Q - 1) + \frac{1}{2\beta_0^2} (\beta_0 Q - 1)^2 + \dots \right], \quad (31)$$

$$W = \frac{1}{\mathcal{L}} e^{\frac{2}{9}\beta_0} \left[ 1 - \frac{2\beta_0}{9} (\beta_0 Q - 1)^2 + \mathcal{O}(\beta_0 Q - 1)^3 \right]. \quad (32)$$

For  $\Lambda z$  and  $T/\Lambda$  the situation is more subtle: to ensure convergence of the integrals (16) and (20) at the lower limit one must partially integrate in the power of  $\beta_0 Q - 1$  to decrease the exponent. One partial integration gives the leading power. For  $\Lambda z$  one has

$$\Lambda z = [e^{-\frac{1}{\beta_0} - \frac{2}{9}\beta_0} (\beta_0 Q - 1)^{-\frac{1}{\beta_0}} + I_z(\beta_0)] [1 + \mathcal{O}(\beta_0 Q - 1)], \quad (33)$$

where

$$I_z(\beta_0) = \int_{1/\beta_0}^{\infty} \frac{dQ}{(\beta_0 Q - 1)^{1/\beta_0}} \frac{d}{dQ} \left[ \beta_0 Q e^{-Q - \frac{4}{9Q} + \frac{2}{9\beta_0 Q^2}} \right] \quad (34)$$

is an integral arising from partial integration. This converges for  $\beta_0 > 1$ . If  $\beta_0 < 1$  more partial integrations are needed to obtain finite subleading terms. Anyway the leading term (33) implies that the fields always extend to  $z = \infty$  in the IR limit.

For  $T$  the leading term is

$$\frac{T(\beta_0 Q \rightarrow 1)}{\Lambda} = \frac{1}{\pi} e^{\frac{1}{\beta_0} + \frac{2\beta_0}{9}} (\beta_0 Q - 1)^{\frac{1}{\beta_0}} \left[ 1 - e^{\frac{4}{\beta_0} + \frac{2\beta_0}{9}} I_T(\beta_0) (\beta_0 Q - 1)^{\frac{4}{\beta_0}} \right], \quad (35)$$

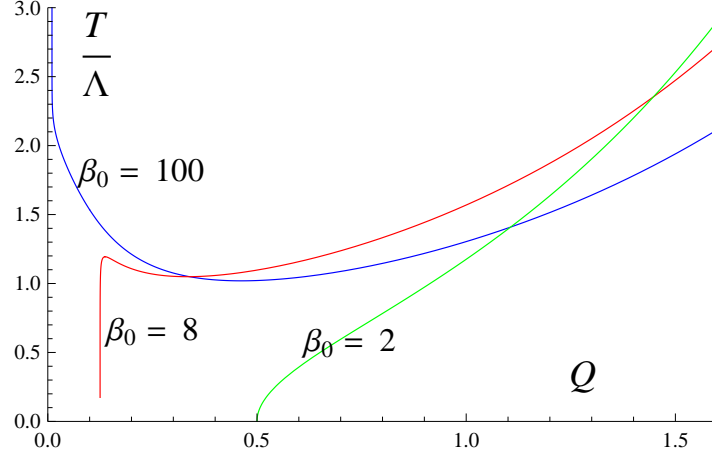


Figure 1: Temperature vs.  $Q = 1/(\beta_0 \lambda)$  as computed from (20). The minimum disappears for  $\beta_0 < 6.58$ . Always  $T(Q = 1/\beta_0) = 0$ , see Eq.(35).

where

$$I_T(\beta_0) = \int_{1/\beta_0}^{\infty} \frac{dQ}{(\beta_0 Q - 1)^{4/\beta_0}} \frac{d}{dQ} \left[ \beta_0 Q e^{-4Q - \frac{4}{9Q} + \frac{2}{9\beta_0 Q^2}} \right], \quad \beta_0 > 4, \quad (36)$$

converges if  $\beta_0 > 4$ . Using this in (26) one obtains for the sound velocity in the conformal region

$$c_s^2 = \frac{1}{3} \left[ 1 - 4e^{\frac{4}{\beta_0} + \frac{2\beta_0}{9}} I_T(\beta_0) (\beta_0 Q - 1)^{\frac{4}{\beta_0}} \right]. \quad (37)$$

For the pressure the next-to-leading order expansion in the IR conformal domain is (see Figs. 3 and 4)

$$\frac{p}{T^4} = \frac{\mathcal{L}^3}{4G_5} \frac{\pi^3}{4} e^{-\frac{2\beta_0}{3}} \left[ 1 + \frac{3}{2} e^{\frac{4}{\beta_0} + \frac{2\beta_0}{9}} I_T(\beta_0) (\beta_0 Q - 1)^{\frac{4}{\beta_0}} + \mathcal{O}(\beta_0 Q - 1) \right]. \quad (38)$$

The overall magnitude here is the same as at large  $T$  but suppressed by the factor  $e^{-\frac{2\beta_0}{3}}$ .

## 4 The two phases

To clarify the structure of the solution, the behavior of  $T(Q)$  for a few values of  $\beta_0$  is shown in Fig. 1. For any value of  $\beta_0$ ,  $T$  vanishes  $T(Q)/\Lambda \sim (\beta_0 Q - 1)^{1/\beta_0}$  at the infrared endpoint, see Eq. (35). For  $Q \gg 1/\beta_0$  the limit is  $T(Q)/\Lambda \sim e^Q$ , Eq. (27). In between,  $\beta_0 = 6.58$  separates two qualitatively different behaviors. For  $\beta_0 < 6.58$   $T(Q)$  grows monotonically with  $Q$ ,  $dT/dQ > 0$  always. For  $\beta_0 > 6.58$ ,  $T(Q)$  has a maximum and minimum, i.e., two zeroes of  $dT/dQ$  and a range where  $dT/dQ < 0$ ,  $c_s^2 < 0$  so that the system is mechanically unstable.

To see concretely what this implies for physics, consider separately the two cases  $\beta_0 = 8$  with two zeroes of  $dT/dQ$  and  $\beta_0 = 2$  with  $dT/dQ > 0$ .

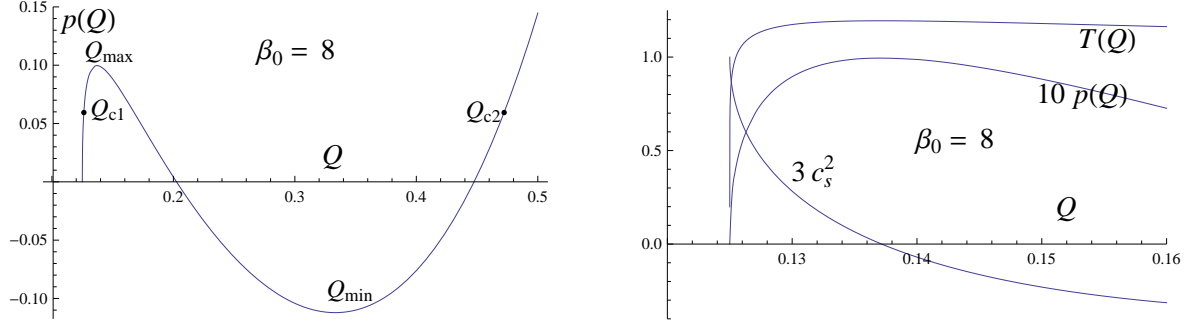


Figure 2: (Left panel) The pressure for  $\beta_0 = 8$ . Note the positive peak at  $Q = 0.137$ , caused via Eq. (23) by the increase of  $T(Q)$  starting from the IR limit  $1/\beta_0 = 0.125$ . (Right panel) Field configurations in the conformal region for  $\beta_0 = 8$ . Note that the sound velocity vanishes at the same point  $Q = Q_{\max} = 0.137$  where  $p'(Q)$  and  $T'(Q)$  vanish.

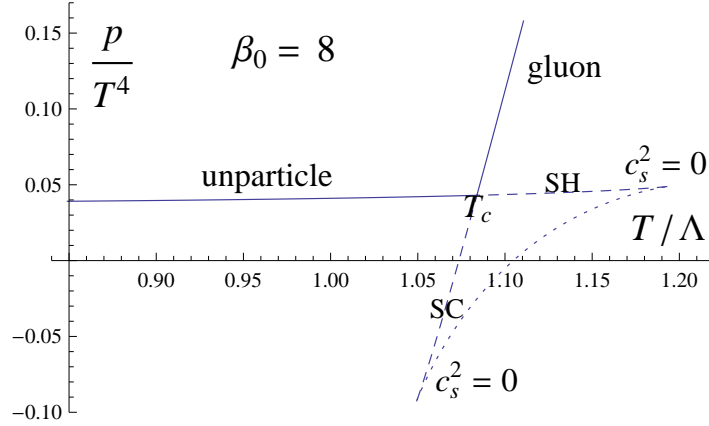


Figure 3: Plot of  $p/T^4$  vs  $T/\Lambda$  in the critical region for a non-monotonic  $T(Q)$ . There is a stable high  $T$  gluon plasma phase with a metastable supercooled (SC) branch and a stable low  $T$  unparticle phase with a metastable superheated (SH) branch. The metastable branches terminate at the points where sound velocity squared becomes negative; between these points the system is mechanically unstable. There is a first order phase transition at  $T_c = 1.084\Lambda$ . The unparticle phase extends to  $T = 0$  to the value  $\pi^3 e^{-16/3}/4 = 0.037$ , the gluon plasma phase at  $T \gg T_c$  to  $\pi^3/4 = 7.75$ . For normalisation to ideal gas, multiply by the factor in Eq. (30).

Fig. 2 shows for  $\beta_0 = 8$  a plot of  $p(Q)$  and a blow-up of the far infrared region. The two zeroes of  $T'(Q)$  correspond to the two zeroes of  $p'(Q)$ , a maximum and a minimum of  $p(Q)$ . The physics is seen from the plot of pressure vs.  $T$  in Fig. 3. The correspondence between Figs. 2 and 3 is as follows:

- The range  $1/\beta_0 < Q$  up to the first maximum  $Q_{\max}$  of  $p(Q)$ , at which  $c_s^2 = 0$ , corresponds to the unparticle phase in Fig. 3. Only the part of this up to  $Q_{c1}$  is thermodynamically stable, the rest is a metastable superheated branch.

- The interval  $Q_{\max} < Q < Q_{\min}$  in which  $p(Q)$  decreases in Fig. 2 corresponds to the unstable phase with  $c_s^2 < 0$  in Fig. 3.
- The interval of  $Q > Q_{\min}$  in which  $p(Q)$  again increases in Fig. 2 corresponds to the quark-gluon plasma phase; only the part of this above  $Q_{c2}$  is thermodynamically stable, the rest is a metastable supercooled branch.
- The phase transition takes place at  $T_c = 1.084\Lambda$ . The latent heat is numerically

$$\frac{\Delta\epsilon}{T_c^4} = \frac{\mathcal{L}^3}{4G_5} 4.55, \quad (39)$$

where (30) gives the normalisation. A fit to nearby values is  $L/T_c^4 = 3.86(\beta_0 - 6.58)^{0.427}$ .

If one now decreases  $\beta_0$ , the first order transition in Fig. 3 becomes weaker and terminates at  $\beta_0 = 6.582$  at  $T = 0.991\Lambda$  and  $p/T^4 = 0.134$  (which decreases to 0.0963 at  $T = 0$  for this  $\beta_0$ ). Below that the transition is continuous. The situation for  $\beta_0 = 2$  is shown in Fig. 4. Since  $dT/dQ > 0$  there is no range with negative  $c_s^2$  and the unstable phase in Fig. 3 disappears. Related to this, there is no discontinuity in  $p'(T)$ , the two phases are continuously connected. Fig. 4 also shows the interaction measure

$$\frac{\epsilon - 3p}{T^4} = T \frac{\partial}{\partial T} \frac{p}{T^4}. \quad (40)$$

Its maximum at  $T = 1.81\Lambda$  could be used as an estimate of the crossover temperature at this  $\beta_0$ .

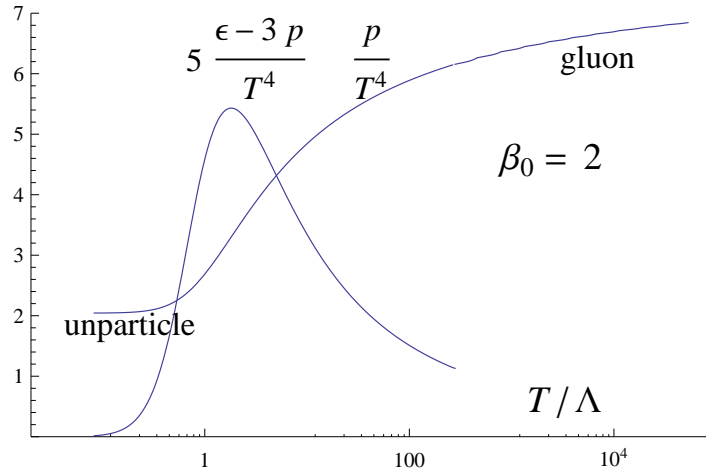


Figure 4: Plot of  $p/T^4$  and of the interaction measure  $(\epsilon - 3p)/T^4$  (multiplied by 5) vs  $T/\Lambda$  for  $\beta_0 = 2$  with a monotonic  $T(Q)$ . The two phases are continuously connected. The unparticle phase extends to  $T = 0$  to the value  $\pi^3 e^{-4/3}/4 = 2.043$ , the gluon plasma phase at  $T \gg \Lambda$  to  $\pi^3/4 = 7.75$ . For normalisation to ideal gas, multiply by the factor in Eq. (30).

## 5 Conclusions

We have, in this article, studied the thermodynamics of a field theory with the beta function (1) with an infrared fixed point using a gauge/gravity duality model [1, 2, 3]. In the two limits  $T = 0$  and  $T = \infty$  we have conformal theories with pure  $\text{AdS}_5$  spaces as gravity duals. The crucial parameter  $\beta_0$  of this model is essentially the ratio of the radii of these spaces, Eq. (2). Varying  $T$  drives a phase transition which in this model is of first order for  $\mathcal{L}_{\text{UV}}/\mathcal{L}_{\text{IR}} > 4.32$ , continuous otherwise.

The computations here used Eqs.(6)-(8) to determine the metric functions, scalar field, and the phase structure [3]. However, we have also verified that the same phase structure is obtained by integrating numerically the Einstein equations (5)-(7) with the scalar potential in (14) and by computing the equation of state as in [2]. Quantitative details change somewhat, for example, the critical value of  $\beta_0 \lambda_*$  is 4.85 instead of 6.58 above.

It has been a very challenging task to identify four dimensional gauge field theories with beta functions of the type (1). These need fermions in special representations [6] and the proof requires extensive numerical effort [10, 11, 12, 13, 14, 15, 16]. It is clear that the finite temperature properties of these theories will be studied but the path to clarification of the phase structure will be long.

Theoretically it will be interesting to study the finite  $T$  phase structure of theories with a walking technicolor beta function, for which the IR fixed point is only approached. The theory then is confining and there is no conformal unparticle phase at  $T = 0$ . The model in this article can be used for that purpose.

*Acknowledgements.* We thank K. Rummukainen, V. Suur-Uski and K. Tuominen for discussions and F. Nitti for the well documented numerical code used in [2]. JA thanks the Magnus Ehrnrooth foundation for financial support.

## References

- [1] U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti, “Holography and Thermodynamics of 5D Dilaton-gravity,” arXiv:0812.0792 [hep-th].
- [2] U. Gursoy, E. Kiritsis, L. Mazzanti and F. Nitti, “Improved Holographic Yang-Mills at Finite Temperature: Comparison with Data,” arXiv:0903.2859 [hep-th].
- [3] J. Alanen, K. Kajantie and V. Suur-Uski, “A gauge/gravity duality model for gauge theory thermodynamics,” Phys. Rev. D **80**, 126008 (2009) [arXiv:0911.2114 [hep-ph]].
- [4] U. Gursoy and E. Kiritsis, “Exploring improved holographic theories for QCD: Part I,” JHEP **0802**, 032 (2008) [arXiv:0707.1324 [hep-th]].
- [5] U. Gursoy, E. Kiritsis and F. Nitti, “Exploring improved holographic theories for QCD: Part II,” JHEP **0802**, 019 (2008) [arXiv:0707.1349 [hep-th]].

- [6] F. Sannino and K. Tuominen, “Techniorientifold,” *Phys. Rev. D* **71**, 051901 (2005) [arXiv:hep-ph/0405209].
- [7] T. A. Rytov and F. Sannino, “Supersymmetry Inspired QCD Beta Function,” *Phys. Rev. D* **78**, 065001 (2008) [arXiv:0711.3745 [hep-th]].
- [8] M. Jarvinen and F. Sannino, “Holographic Conformal Window - A Bottom Up Approach,” arXiv:0911.2462 [hep-ph].
- [9] O. Antipin and K. Tuominen, “Constraints on Conformal Windows from Holographic Duals,” arXiv:0912.0674 [hep-ph].
- [10] T. Appelquist, G. T. Fleming and E. T. Neil, “Lattice Study of the Conformal Window in QCD-like Theories,” *Phys. Rev. Lett.* **100**, 171607 (2008) [Erratum-ibid. **102**, 149902 (2009)] [arXiv:0712.0609 [hep-ph]].
- [11] S. Catterall, J. Giedt, F. Sannino and J. Schneible, “Phase diagram of SU(2) with 2 flavors of dynamical adjoint quarks,” *JHEP* **0811**, 009 (2008) [arXiv:0807.0792 [hep-lat]].
- [12] T. DeGrand, Y. Shamir and B. Svetitsky, “Phase structure of SU(3) gauge theory with two flavors of symmetric-representation fermions,” *Phys. Rev. D* **79**, 034501 (2009) [arXiv:0812.1427 [hep-lat]].
- [13] A. J. Hietanen, K. Rummukainen and K. Tuominen, “Evolution of the coupling constant in SU(2) lattice gauge theory with two adjoint fermions,” *Phys. Rev. D* **80**, 094504 (2009) [arXiv:0904.0864 [hep-lat]].
- [14] L. Del Debbio, B. Lucini, A. Patella, C. Pica and A. Rago, “Conformal vs confining scenario in SU(2) with adjoint fermions,” *Phys. Rev. D* **80**, 074507 (2009) [arXiv:0907.3896 [hep-lat]].
- [15] Z. Fodor, K. Holland, J. Kuti, D. Negradi and C. Schroeder, “Chiral properties of SU(3) sextet fermions,” *JHEP* **0911**, 103 (2009) [arXiv:0908.2466 [hep-lat]].
- [16] D. K. Sinclair and J. B. Kogut, “QCD thermodynamics with colour-sextet quarks,” arXiv:0909.2019 [hep-lat].